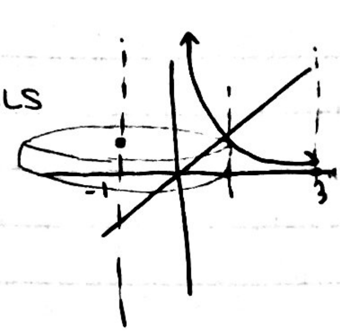
MAT1322
Lecture 22

Review Part 2

VOLUMES VIA CYLINDRICAL SHELLS



Ex $y = x$, $y = \frac{1}{x}$ $\Big\}$ between $x=1$ and $x=3$

rotate around $y=-1$ axis

POI:
$x = \frac{1}{x}$
$x = 1$

Area of a cylinder at x: circumference × height

$A = \underbrace{2\pi(x-(-1))}_{\text{circumference}}\underbrace{(x - \frac{1}{x})}_{\text{height}}$ ← find formula for cross section area

$V = \int_1^3 A(x)\,dx$ ← integrate over cross-section area

$= \int_1^3 2\pi(x+1)(x-\frac{1}{x})\,dx$

$= 2\pi \int_1^3 (x^2 - 1 + x - \frac{1}{x})\,dx$

$= 2\pi \left[\frac{x^3}{3} - x + \frac{x^2}{2} - \ln(x)\right]_1^3$

$= 2\pi\left[\frac{27}{3} - 3 + \frac{9}{2} - \ln(x) - \frac{1}{3} + 1 - \frac{1}{2} + \cancel{\ln(1)}^{\,0}\right]$

$= 2\pi\left[6 + \frac{9}{2} - \ln(3) - \frac{1}{3} + \frac{1}{2}\right]$

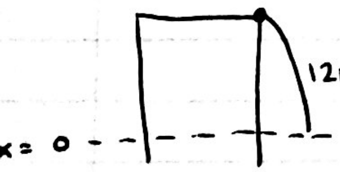
$= 2\pi\left[11 - \frac{1}{3} - \ln(3)\right]$

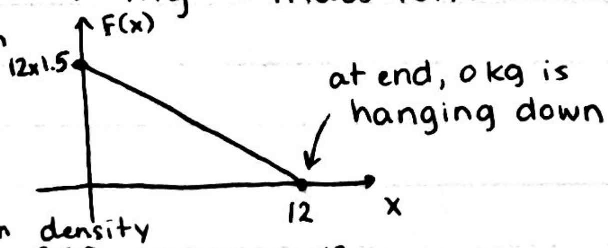
$= \underline{2\pi\left[-\ln(3) + \frac{32}{3}\right]}$.

WORK Rope: 12m, hanging over a building
density: 1.5 kg/m
Calculate work needed to pull it up

$W = F \cdot d$ (d: displacement)
↳ works if force is constant

$F = mg$ → mass isn't constant!





Force is not constant because mass decreases.

$F(x) = \underbrace{(12 - x)}_{\text{length hanging down}} \cdot \underbrace{1.5}_{\text{density}}\, g$

$W = \int_0^{12} (12-x)(1.5)(9.8)\,dx$

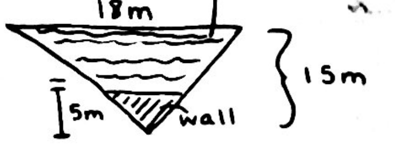
$= 14.7 \int_0^{12} (12-x)\,dx$

$= 14.7\left[12x - \frac{x^2}{2}\right]_0^{12}$

$= 14.7\left[144 - \frac{144}{2}\right]$

$= \underline{1058.4\ J}$

HYDROSTATIC FORCE

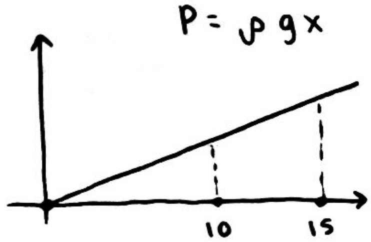


Compute the hydrostatic force on the wall

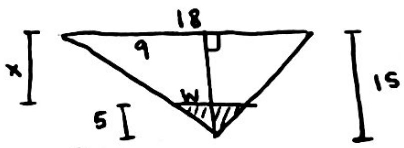
$F = P \cdot A$, $P = \rho g d$ ← depth (*varies)   top of wall has depth 10 m $\int_{10}^{15}$

area of rectangles   density   gravity   bottom has depth 15 m

$P = \rho g x$



A: We need an area for the force. For this we need the width of the wall, which depends on the depth!



½ width at depth x: $\frac{9}{15} = \frac{w}{15-x}$

$w = \frac{9}{15}(15-x) = 9 - \frac{9x}{15}$

total width: $\underline{\underline{2w = 18 - \frac{18x}{15}}}$

500 N $F = \int_{10}^{15} \overbrace{\rho g x}^{\text{pressure}} \cdot \underbrace{\left(18 - \frac{18x}{15}\right)}_{\text{width}} \underbrace{dx}_{\text{change in depth}} = 2\cdot 857500 \text{ N}$

MACLAURIN SERIES (Taylor series at a=0)

(a) Find MacLaurin series of e^{-x^2}

(b) Use this to give a MacLaurin series for $\int_0^x e^{-t^2}\,dt$

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a) recall: $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ (for all x)

To get the MacLaurin series for e^{-x^2}, we substitute:

$$e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{n!}$$

b) if $f(x) = \int_0^x e^{-t^2}\,dt = \int_0^x \left(\sum_{n=0}^{\infty} \frac{(-1)^n t^{2n}}{n!} \right) dt$

$$= \sum_{n=0}^{\infty} \int_0^x \frac{(-1)^n t^{2n}}{n!}\,dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^x t^{2n}\,dt$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left[\frac{t^{2n+1}}{2n+1} \right]_0^x = \underline{\sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \cdot \frac{x^{2n+1}}{2n+1}}$$